

A critical point is an x -value in the domain of $f(x)$ for which $f'(x) = 0$ or $f'(x) = \text{DNE}$.

$$\text{ex } f(x) = \frac{2x^2}{x-2}$$

$$\textcircled{1} f'(x) = \frac{(x-2) \cdot (4x) - (2x^2) \cdot (1)}{(x-2)^2} = \frac{2x^2 - 8x}{(x-2)^2}$$

$$\textcircled{2} \text{ Set } f'(x) \text{ equal to zero and solve for } x \text{ (Horizontal Tangent Lines)}$$

$$\frac{2x^2 - 8x}{(x-2)^2} = 0 \rightarrow 2x(x-4) = 0$$

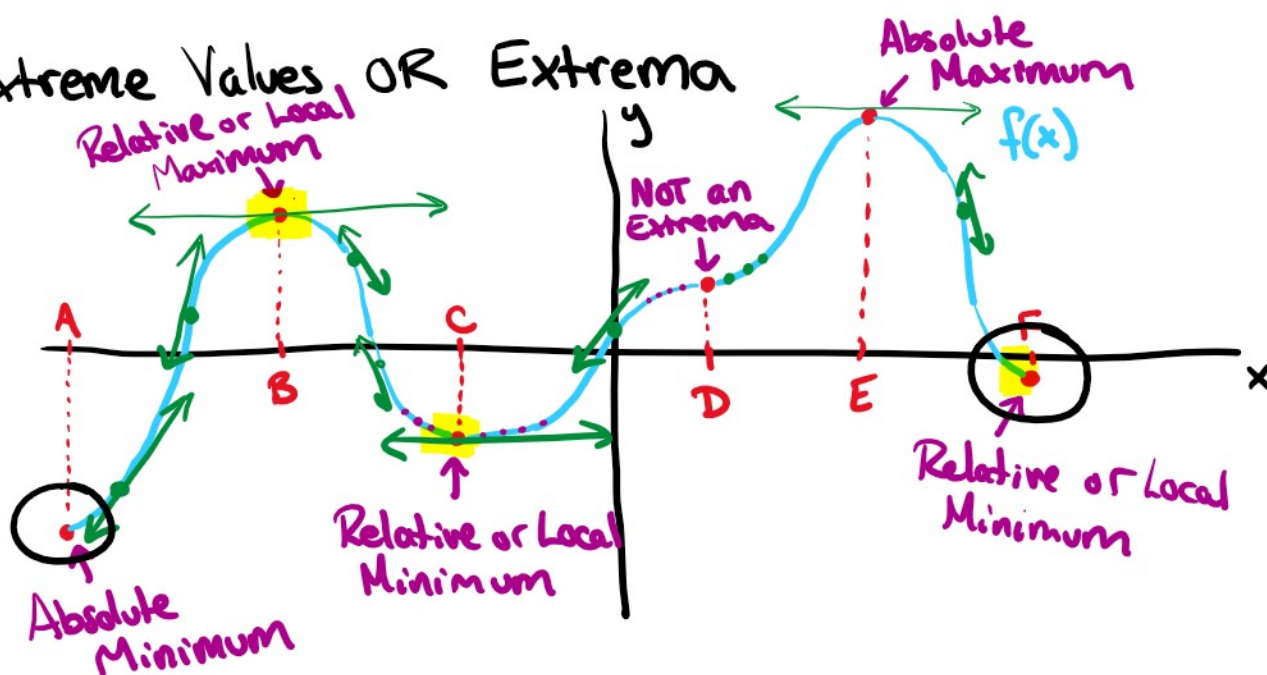
$$x=0 \quad x=4$$

$$\textcircled{3} \text{ Identify any } x\text{-values that make } f'(x) \text{ DNE}$$

$$\frac{2x^2 - 8x}{(x-2)^2} = 0 \rightarrow x=2$$

Critical Points: $x = \underline{0, 2, 4}$

Extreme Values OR Extrema



* Extreme Values only occur at Critical Points *

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and Interval Endpoints

ex $f(x) = (x^2 - 25)^2$ on interval $[-2, 8]$

① $f'(x) = 2(x^2 - 25)' \cdot (2x)$

② Set $f'(x)$ equal to zero: $4x(x^2 - 25) = 0$

③ Identify where $f'(x)$ DNE $\left\{ \begin{array}{l} 4x(x+5)(x-5) = 0 \\ \downarrow \quad \downarrow \quad \downarrow \\ x=0 \quad x=-5 \quad x=5 \end{array} \right.$

Critical Points: $x = 0, -5, 5$ outside of given interval

④ Plug each critical point AND each interval endpoint into $f(x)$ to find their corresponding y -values

$x=0$ ~~$x=-5$~~
 $y=625$
 $(0, 625)$

Relative
Max/Min

$x=5$
 $y=0$
 $(5, 0)$
Absolute
Minimum
(least y -value)

$x=-2$
 $y=441$
 $(-2, 441)$
Relative
Max/Min

$x=8$
 $y=1521$
 $(8, 1521)$
Absolute
Maximum
(greatest y -value)